Research on Learning Method Based on Linear Space Concept

Hongxin Bai

School of Data Science and Software Engineering, Baoding University, Baoding, Hebei, 071000, China

Keywords: linear space; Learning method; Integrity; Sports thought

Abstract. When learning the concept of linear space, students have difficulty in understanding the abstraction of vector and the integrity of the system. The learning methods were explained from the micro and macro perspectives, so that students could grasp the five dimensions from mathematics learning. This work aimed to make students better understand the concept of linear space from the micro and macro perspectives. Also, some academic suggestions were put forward from the integrity of the system structure and the idea of movement contained in the concept.

1. Introduction

In the process of higher algebra learning, many students think that the concept of linear space is an insurmountable threshold for further higher algebra learning. The reason is that most students are difficult to understand the abstract concept for higher algebra and rely on digital operation in middle school algebra. They can't understand the operation and operation law of text from the previous operation in multinomial, vector, matrix, etc., and they also can't understand the meaning of structure and system. At the same time, their learning still only focuses on the operation, but does not pay attention to the operation law level. Students can't understand a mathematical problem as a whole, but can only analyze some elements from the system. Some details from the system are emphasized, while the relationship between elements and the overall structure from the system are not emphasized. If the concept for linear space is not well learned, the students will form certain obstacles to the future linear transformation, Euclidean space, etc., as well as the learning of recent algebra. It also directly affects their interest and belief in learning higher algebra, so that they can't better grasp the five dimensions of mathematics learning, including epistemology dimension, self-dimension, knowledge transfer dimension, learning behavior dimension and learning psychological dimension.

2. Actively Recalling the Intuitive Model Related to the Concept Based on Teachers' Guidance from Epistemology Dimension

The basic property of the three-dimensional space and the vector in space discussed in analytic geometry is that the vector can be added according to the parallelogram rule, or it can be multiplied quantitatively with the real number. For a set of matrices, two identical matrices can be added, and a real number can be multiplied by a matrix. These are helpful for us to understand the concepts for the abstract vector addition and the number multiplication vector. At the same time, the concepts for these intuitive models are quite different from the abstract mathematical concepts. From the epistemology dimension, it is necessary to learn to eliminate the special elements of specific concepts and pay attention to some common properties of specific concepts. The dialectical materialist cognitive line of "cognition-practice" should be followed, and the methods of observation, experiment and induction should be applied. The research object of space is an abstract vector. Since people's ability for spatial imagination is limited, they can go further by virtue of algebra reasoning ability. With the help of Descartes' philosophical research from the epistemology dimension, the purpose on the research is to seek the laws of nature. Therefore, we must first find a unified method to achieve this goal. Any problem in the natural world can be transformed into a mathematical problem, and the mathematical problem can always be transformed into an algebraic problem. By introducing the algebraic method, the geometric problems, polynomials, matrices, etc.,

  DOI: 10.25236/ic3er.2019.022

can be operated according to the fixed rules, which is the real purpose of understanding space.

3. Incorporating the Learning of New Concepts into the Already Familiar Concept System, Examining the Position and Role of New Concepts in the Conceptual System, Familiarizing it with the Relationships and Differences between Similar Concepts, and Reunderstanding the New Concepts from Self Dimension

For example, in the concept of linear space, n ary ordered array $(a_1, a_2, \cdots, a_n)$ as the n dimensional vector space of elements can refer to the previous discussion. For n ary ordered array, there are addition and quantity multiplication, that is

$$(a_1, a_2, \cdots, a_n) + (b_1, b_2, \cdots, b_n) = (a_1 + b_1, a_2 + b_2, \cdots, a_n + b_n)$$

$$k(a_1, a_2, \cdots, a_n) = (ka_1, ka_2, \cdots, ka_n)$$

Meanwhile, the two operations of n dimensional vector meet the following operation rules:

(1) $\alpha + \beta = \beta + \alpha$ (2) $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ (3) α **+0=+** α (4) $\alpha + (-\alpha) = 0$

(5) $k(\alpha + \beta) = k\alpha + k\beta$ (6) $(k + l)\alpha = k\alpha + l\alpha$ (7) $k(l\alpha) = (kl)\alpha$ (8)$1\alpha = \alpha$

It can be seen that the above operations and operation rules are exactly the concept of linear space applied to n dimensional vector space. However, these operations and operation rules can also be applied to other systems, such as real numbers, complex numbers, matrices, linear equations, etc. That is to say, the concept of linear space has the same algebraic structure belonging to different systems.

The self-dimension is a process in which students can deduce the unknown through their familiar things, reveal the meaning of mathematical facts, and express their unique individual understanding on mathematics. The self-dimension of comprehension needs to reveal the meaning of mathematical facts. By describing the meaning for mathematical facts, students' understanding and insight into the spatial operation, a specific mathematical object, can be facilitated. The meaning of many mathematical objects is always hidden behind the facts, which is always abstract and imperceptible. Generally speaking, it is difficult for students to understand this abstract and obscure information. The purpose of the self-dimension is to transform the representation of mathematical objects. The meaning behind mathematical objects is found and revealed through comparison, analogy, induction, deduction, analysis, synthesis and other processing methods, in order to achieve this dimension of understanding. At the same time, self-reflection is also an important carrier of self-dimension, which is a crucial dimension on mathematical understanding, an abstract thinking activity and constant repeated thinking. It requires students to think about the process of mathematics learning and thinking activities, to think and comprehend the knowledge, methods, ideas and strategies involved in mathematics activities, thus extracting experience and summarizing lessons. So that the cognition is gradually deepened or the starting point of new thinking problems is obtained, and the purpose of adjusting and improving the mathematical cognitive structure is achieved. Reflection can also rationally recognize the advantages and limitations for their own thinking and behavior patterns, and can clearly identify and avoid misunderstandings caused by their own prejudices, habits, thinking or behavior patterns. Reflection requires students to answer the following questions constantly. How do I think? What mathematical examples, thoughts, methods and skills have been used? What are the limitations of my understanding? What are my shortcomings? What are my misunderstandings? What kind of analogy should I make? Thus, the essence of mathematical theory is further insight, and the essence of mathematical thought is understood to achieve a high level of understanding.

4. The Definition of Linear Space from Knowledge Transfer Dimension Should be Carefully Considered and Its Exact Meaning Should be Grasped

Applied knowledge transfer is the third dimension in mathematical understanding. Applied knowledge transfer in mathematical understanding refers to the ability to use the mathematical knowledge to carry on the creative thinking, and put forward the innovative thinking method and the skilled mathematics skill to solve the problem in the concept study. In the application process, students should answer the following questions. Where and how to use the learned mathematical knowledge and skills? First of all, understanding of applied knowledge transfer needs to combine mathematical knowledge with specific examples, so that the two are consistent. Here, the so-called applied knowledge transfer is different from the simple application in mathematical knowledge itself or mathematical knowledge. To achieve the application of comprehension, students can apply knowledge without any help from any hint, and they are faced with problems that are new or real concepts. For example, in the concept of linear space, an algebraic operation is defined between the elements of set V, which is called "an algebraic operation" in "addition". It means a corresponding law (i.e., a mapping or transformation, rather than an ordinary addition operation of numbers). Second, zero vector, negative vector and unit "1" should also be paid enough attention.

Example 1, the set R^+ of all positive real numbers, for addition and quantity multiplication $a \oplus b = ab$ and $k \circ a = a^k$, constitutes a vector space on R. Then the zero vector of this space is 0, and the negative vector of $a \in R^+$ is $\frac{1}{a}$. Some students mistakenly think that the zero vector is 0 and the negative vector is $-a$. At this point, you just look at the set R^+, 0 and $-a \in R^+$ discussed, which is meaningless.

Example 2, the set R^+ of all positive real numbers, for addition and quantity multiplication $a \oplus b = ab$ and $k \circ a = a^{-k}$, constitutes a vector space on R. Then the unit element "1" = -1 in this space. In this problem, some students mistakenly believe that it cannot constitute a linear space on R.

The above situation is unable to accurately grasp the performance of conceptual knowledge transfer. Secondly, the application of understanding also needs to innovate the original mathematical knowledge. Swiss child psychologist Piaget points out that students' understanding is manifested through innovation in their application of knowledge. The application of understanding requires going beyond the conventional way and using a variety of methods and strategies to solve the problem from many perspectives. In fact, only by recreating mathematical knowledge can students really internalize knowledge, understand it for themselves, and show flexibility in applying knowledge to new situations and solving practical problems in behavior.

Such as binary sequence of all real numbers, for the operation defined below:

$$(a_1, b_1) \oplus (a_2 + b_2) = (a_1 + a_2, b_1 + b_2 + a_1 a_2)$$

$$k \circ (a_1, b_1) = (ka_1, kb_1 + \frac{k(k-1)}{2} a_1^2$$

Solution: it is not difficult to verify that the addition, commutative law, associative law are satisfied. According to the addition defined by it, (0, 0) is zero, and any (a, b) negative element is $(-a, a^2 - b)$. Meanwhile, for number multiplication:

$1\circ\ (a, b) = (1\circ\ a, 1\circ\ b = \frac{1(1-1)}{2}a^2) = (a,b),$

$k.(l.(a,b) = k.(la, lb + \frac{l(l-1)}{2}a^2) = (kla, k[lb + \frac{l(l-1)}{2}a2] + \frac{k(k-1)}{2}(la)^2)$

$= (kla, k[lb + \frac{l(l-1)}{2}a^2] + \frac{k(k-1)}{2}(la)^2) = (kla, \frac{kl(kl-1)}{2}a^2 + \frac{k(k-1)}{2}(la)^2)$

$= (kla, \frac{kl(kl-1)}{2}a^2 + klb) = (kl).(a,b),$

$(k+l).(a,b) = [(k+l)a, \frac{(k+l)(k+l-1)}{2}a^2 + (k+l)b]$

$k.(a,b) \oplus l.(a,b) = (ka, kb + \frac{k(k-1)}{2}a^2) \oplus (la, lb + \frac{l(l-1)}{2}a^2$

$= (ka + la, kb + \frac{k(k-1)}{2}a^2 + \frac{k(k-1)}{2}a^2 + kla^2)$

$= [(k+l)a, \frac{(k+1)(k+l-1)}{2}a^2 + (k+l)b].$

That is $(k+l) \circ (a,b) = k \circ (a,b) \oplus l \circ (a,b)$.

$k \circ [(a_1, b_1) \oplus (a_2, b_2)] = k \circ (a_1 + a_2, b_1 + b_2 + a_1 a_2)$

$= [k(a_1 + a_2), k(b_1 + b_2 + a_1 a_2 + \frac{k(k-1)}{2}(a_1 + a_2)^2)]$,

$k \circ (a_{1,} b_1) \oplus k \circ (a_2, b_2)$

$= (ka_1, kb_1 + \frac{k(k-1)}{2}a_1^2) \oplus (ka_2, kb_2 + \frac{k(k-1)}{2}a_2^2)$

$= (ka_1 + ka_2, kb_1 + \frac{k(k-1)}{2}a_1^2 + kb_2 + \frac{k(k-1)}{2}a_2^2 + k^2 a_1 a_2)$

$= (k(a_1 + a_2), k(b_1 + b_2 + a_1 a_2) + \frac{k(k-1)}{2}a_1^2 + + \frac{k(k-1)}{2}a_2^2 + k^2 a_1 a_2 - k\ a_1 a_2)$

$= (k(a_1 + a_2), k(b_1 + b_2 + a_1 a_2) + \frac{k(k-1)}{2}(a_1^2 + a_2^2)^2)$,

That is $k \circ (a_1, b_1) \oplus (a_2, b_2) = k \circ (a_{1,} b_1) \oplus k \circ (a_2, b_2)$. Therefore, the given set constitutes a linear space.

5. The Concept of Linear Space Should be Grasped as a Whole from Learning Behavior Dimension

The learning behavior dimension is the fourth dimension in the process of mathematics learning. Students can deeply understand mathematics knowledge through critical insight, so that they can see its inner essence and the whole through the surface of the problem, and give some conventional mathematical ideas new meaning. This is an innovative understanding that is the ability to get through a certain amount of accumulation. Gardner believes that deep understanding lies in presenting the important characteristics of a problem in different ways and solving it in different ways from multiple perspectives. Insightful students are often able to notice details that are often dismissed and overlooked by others, making it easier to look critically at each issue. They often ask the question: how do you look at this mathematical problem from an overall point of view? Is there any other form of change in this mathematical problem? Is the conclusion of this question correct and can the conclusion be popularized? The holistic structure we advocate in mathematics learning can cultivate students' critical insight.

For example, students only know to check the eight operation rules correctly according to the given topic, which is considered to be a success when learning the concept of linear space. As a result, they don't know the linear space and only remember eight operation rules, which is a typical learning method that only sees trees but not forests. If learning mathematics is only limited to microlearning, the real mathematical ideas and methods can’t be learned, and the application of mathematical knowledge to creatively solve practical problems is even less. The habit of considering problems in an all-round way should be cultivated. We should not only see a part of the mathematical concept, but also see the relationship between the whole and the part, thus avoiding one-sidedness when studying problems.

6. Understanding the Movement Thought in the Concept of Linear Space from Learning Psychological Dimension and Grasping the Concept with the Thought of Dialectics

The learning psychological dimension is the fifth dimension in mathematical learning, which refers to the ability to deeply understand the movement and dialectics of concepts. Mathematical understanding involves not only cognition, but also emotional movement and discrimination. In this sense, we must do two things to achieve understanding in mathematics learning. First, in the process of concept change, it is reasonable to transfer the mathematical problems psychologically. At the same time, any dialectical thought and method of solving mathematical problems should also be known. Each method of thinking has enlightenment, which can broaden students' horizons and broaden their perspectives on problems. The learning psychological dimension in mathematics understanding reflects the social characteristics of mathematics learning. It reflects students’ support and mutual assistance in the process of constructing their understanding of mathematics. This power is rooted in learners' reflection and deep insight into other people's views, and based on this; they make psychological reflection and improvement on their own experience. Second, it is necessary to deeply understand the psychological process of dialectics, contradiction, perplexity and success experienced by mathematicians in the process of mathematical creation and invention. The implicit innovation charm and value behind the development of important mathematical thoughts should be deeply felt, thus reaching a deep understanding beyond the surface understanding.

Like the concept of functions, the concept of linear space embodies the dependence between changing processes and varying quantities. Meanwhile, it guides some specific change processes and variables, which are not related to which special thing. Linear space first introduces abstract variable vector into higher algebra, and with variables, motion enters the concept of linear space. The whole concept embodies the mutual penetration of motion and change, and lays a good foundation for the subsequent abstract mathematics course.

7. Conclusion

The understanding of mathematical concepts requires the coordination and unity between abstract and concrete. Generally speaking, the understanding of mathematical concepts in students' eyes is that they can really understand the conceptual elements, master the essential connotation of concepts, and can use conceptual knowledge to solve practical problems. It can be seen that their so-called conceptual understanding is limited to the specific cognitive level and ignores the abstract perspective. Some studies have shown that even if students give a mathematical problem that seems to be perfect on the surface, it does not show that students fully understand the relevant mathematical concepts as a whole. Some students may just memorize and imitate them. In the process of learning, the movement and change contained in the concept should be emphasized. By understanding and penetrating the changes in mathematics, students can discover the thought process understood by students, the depth of understanding, as well as the original insight. In this way, movement insight and change experience of mathematical concept understanding can be achieved. The concept of linear space is a perfect combination of abstract and concrete, whole and local, motion and change. Beginners must deeply experience and understand the beauty of concepts. Thus, students' interest, self-confidence, willpower, and learning efficiency, etc., in learning advanced algebra and subsequent courses are better cultivated.

References

[1] Zhang Shunyan. From Variable Mathematics to Modern Mathematics [J]. Advanced Mathematics Research, 2006, (5): 2-4; (6): 7-11.

[2] Zhang Miaozhou. Teaching of Calculus: From Cold Beauty to Fiery Thinking [J]. Advanced Mathematics Research, 2006, (2): 2-4.

[3] Cao Dianzhou. Random Talk About How to Teach Mathematics (2) [J]. Advanced Mathematics Research, 2006, (2): 5.

[4] Li Shangzhi. Introducing the Concept of Linear Algebra from the Problem [J]. Advanced Mathematics Research, 2006, (5): 6-8.

[5] Department of Algebra and Geometry, Peking University. Higher Algebra (Third Edition) [M]. Beijing: Higher Education Press, 2003.

[6] Carlson, M. P. The Mathematical Behavior of Six Successful Mathematics Graduate Students Influences Leading to Mathematical Success. Educational Studies in Mathematics, 1999, 40: 237-258.